

Abstract

This thesis investigates several geometric problems concerning rational homogeneous spaces and their linear sections. The study focuses on two main aspects.

In the first part, we study the classification problem of smooth projective varieties with special \mathbb{C}^* -actions. A \mathbb{C}^* -action on a smooth projective variety X is called Euler-type at a fixed point x if its action on the tangent space $T_x X$ is given by scalar multiplications. We prove that for a smooth projective variety X with Picard number 1, X is isomorphic to an irreducible Hermitian symmetric space of tube type if and only if for a general pair of points (x, y) on X , there exists a \mathbb{C}^* -action on X that is Euler-type at x and whose inverse action is also Euler-type at y . This work is primarily based on the works of Hwang-Mok and Fu-Hwang on prolongations of infinitesimal linear automorphisms of projective varieties.

In the second part, we study the moduli spaces of linear sections of rational homogeneous spaces, mainly investigating two classes of linear sections:

The first class concerns linear sections of subadjoint varieties of type F_4, E_6, E_7, E_8 . It is known that their smooth hyperplane sections are deformation rigid. For smooth codimension two linear sections, we prove that their GIT moduli spaces are naturally isomorphic to the moduli spaces of polarized elliptic curves, which in turn admit natural embeddings into the weighted projective space $w\mathbb{P}(1, 3, 4, 6)$. The proof mainly relies on Bhargava-Ho's work on Hermitian cubes.

The second class involves linear sections of the spinor tenfold \mathbb{S}_5 . It is known that smooth linear sections of codimension ≤ 3 have finitely many isomorphism classes. Building on Kuznetsov's work, we prove that the GIT moduli space of codimension 4 linear sections of \mathbb{S}_5 is isomorphic to the moduli space of Kummer surfaces. Furthermore, we prove there exists a unique quasi-homogeneous smooth codimension 4 linear section, and its automorphism group has finitely many orbits.

Key Words: Rational homogeneous space, linear sections, \mathbb{C}^* -actions, GIT moduli space, Kummer surface